Separable $f(x)\,dx = g(y)\,dy$

$\rightsquigarrow$ just integrate both sides.

Linear $\frac{dy}{dx} + P(x)y = g(x)$

$\rightsquigarrow$ multiply both sides by integrating factor

$$I = e^{\int P dx}$$

and recognize new LHS as $\frac{d}{dx}(\text{something})$.

Exact $A\,dx + B\,dy = 0$ where $A_y = B_x$.

Soln is $f(x,y) = C$ where $f_x \overset{①}{=} A$, $f_y \overset{②}{=} B$.

(solve ① first, then sub into ②, or vice versa.)

Hope for the best Given general DE

$$A\,dx + B\,dy = 0$$

can try to multiply both sides by integrating factor $I(x)$ or $I(y)$ to make it exact.

Solutions by substitutions

- **Homogenous**:

$$A\,dx + B\,dy = 0$$

A, B homogenous of same degree.

Let $y(u,x) = ux$ (or $x = vy$)

$\rightsquigarrow$ get separable DE.

- **Bernoulli**

$$\frac{dy}{dx} + P(x)y = f(x)y^n$$

Let $u(x) = y(x)^{1-n}$.

$\rightsquigarrow$ get linear DE.

- **special form**

$$\frac{dy}{dx} = f(Ax + Bx + C)$$

Let $u = Ax + Bx + C$.

$\rightsquigarrow$ get separable DE.